Math 3063 Abstract Algebra Project 4 Name:
Prof. Paul Bailey April 18, 2007

Complete these problems and turn in the solution by Monday, March 19, 2007. Attach this page to the front of the solutions. Solutions should be self explanatory and written in complete sentences.

Cyclic Groups and Dihedral Groups

Problem 1. Let G be a group and let $g \in G$. Suppose that $g^i = g^j$ for some distinct positive $i, j \in \mathbb{Z}$. Let k be the smallest positive integer such that $g^k = g^j$ for some $0 \leq j < k$. Show that $g^k = 1$.

Problem 2. Let G be a group and let $g \in G$ with $\text{ord}(g) = n$.
Let $d, m \in \mathbb{Z}$ be positive such that $\gcd(m, n) = d$. Show that $\text{ord}(g^m) = \frac{n}{d}$.

Problem 3. Let G be a group and let $h, k \in G$ with $hk = kh$.
Show that if $\langle h \rangle \cap \langle k \rangle = \{1\}$, then $\text{ord}(hk) = \text{lcm}(\text{ord}(h), \text{ord}(k))$.

Problem 4. Let $\mathbb{Z}_{18}^* = \{\overline{a} \in \mathbb{Z}_{18} \mid \gcd(a, 18) = 1\}$.

(a) Find the (multiplicative) order of each element in $\mathbb{Z}_{18}^*$.

(b) Construct the Cayley table of $\mathbb{Z}_{18}^*$.

Problem 5. Let $\rho, \tau \in S_5$ be given by $\rho =$ `(1 2 3 4 5)` and $\tau =$ `(2 5)(3 4)`.
The *dihedral group on* 5 *points* is the subgroup of S_5 given by

$$D_5 = \{\epsilon, \rho, \rho^2, \rho^3, \rho^4, \tau, \tau\rho, \tau\rho^2, \tau\rho^3, \tau\rho^4\};$$

view D_5 as the set of rigid motions of a regular pentagon.

(a) Write each element of D_5 in disjoint cycle notation.

(b) Show that $\rho\tau = \tau\rho^4$; describe this geometrically.

(c) Find the lattice of subgroups of D_5.